LBL-10360 c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

ENERGY & ENVIRONMENT DIVISION

To be presented at the 2nd Annual System Simulation and Economic Analysis Conference, San Diego, CA, January 23-25, 1980

PREDICTING THE TIME RESPONSE OF A BUILDING UNDER HEAT INPUT CONDITIONS FOR ACTIVE SOLAR HEATING SYSTEMS

Mashuri L. Warren, Ammar F. Sakkal, and Steven R. Schiller

January 1980

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

PREDICTING THE TIME RESPONSE OF A BUILDING UNDER HEAT INPUT CONDITIONS FOR ACTIVE SOLAR HEATING SYSTEMS*.

Mashuri L. Warren, Ammar F. Sakkal‡, and Steven R. Schiller

Solar Energy Group
Energy and Environment Division
Lawrence Berkeley Laboratory
University of California, Berkeley

ABSTRACT

A dynamic model has been developed to determine the response of a residential building to changes in heat input, and to predict room and air temperature changes. The model has been developed to simulate the response of a typical building for possible use with the LBL test facility that evaluates control strategies for active solar systems. The model must properly predict the rapid rise in room temperature resulting from addition of heat with a fan coil, must give the correct average building load, and must require only limited computational capabilities.

The building is modeled as a three node capacitance-resistance network which can be solved to determine the room air temperature, interior wall temperature, and building-shell wall temperature as a function of time in response to heat input. Capacitance terms represent the thermal masses of the air within the structure, the interior walls, and the interior surface of the outer building shell. Resistance terms represent the resistance to the heat transfer from the interior and exterior walls to the room air, from the exterior wall to the outside, and from the room air to the outside by transmittance through windows and by infiltration. The heat input is provided directly to the air (as, for example, when a heating fan coil is turned on).

---

* This work has been supported by the Systems Analysis and Design Branch, Systems Development Division, Office of Solar Applications, U. S. Department of Energy under Contract No. W-7405-ENG-48.

‡ presently at University of Minnesota.

The time dependent equations are solved using Laplace transforms to determine the building response to a step increase in heat input power. The dynamic response of the building's temperatures can be characterized in terms of the building heat loss coefficient, UA, and three exponential decay terms with appropriate time constants and weights. A time constant of the order of a few minutes describes the air temperature rise due to heat input balanced against losses to the building and to the outside. A time constant of the order of several hours describes the relaxation of the temperature of the building thermal mass to the outside ambient temperature. A third time constant describes the rearrangement of energy between the air and the structure.

The magnitudes of different capacitances and resistances are estimated for a typical residential structure. The time constants and the building heat loss coefficient are then determined from the model. The results are compared with an experimental measurement[1] in which the three time constants have been experimentally determined.

For predicting building temperature response and evaluating control strategies the three node resistance model offers a simple method which can be applied on a small computer and in real time. It is anticipated that this model will provide the thermal accuracy and computational speed suitably matched to the requirements of the LBL test facility for control strategies.

INTRODUCTION

The LBL solar control test facility[2,3] consists of a hydronic solar space heating system with heat input and building load simulator. The load simulator consists of an airflow channel with a heating coil similar to that in the furnace ductwork of a residential heating system.

To make meaningful comparisons between alternate control strategies using identical simulated load conditions, it is necessary to determine the building heating demand and the condition of the building thermostat based on building parameters and weather conditions. Thus an investigation was begun to choose a model to predict average heating requirements and to describe how the room air, wall, and thermostat temperatures respond to sudden changes in heat input when the heating fan turns on or off, and to changing weather conditions. Such modeling becomes of crucial importance if one attempts to simulate the recovery from night thermostat setbacks.

While standard load calculations neglect thermal capacitance effects, some authors[4,5,6] do consider the response of room temperature to sudden heat inputs using a resistance-capacitance thermal network model with a single thermal capacitance term, characteristic of the lumped thermal mass of the building structure. This model gives the temperature response over several hours, which, for instance, can determine the temperature swing and mean temperature of a passively heated structure. Other researchers [7,8,9,10] have considered thermal network models with several nodes.

In our analysis we are constrained to models that can be implemented on our HP 9825A microcomputer which is performing many tasks. We have applied a three node model based on a physical modeling of a building. The exponential response times, and with some modest effort the appropriate weighting factors for the different exponentials are determined. By Laplace transform solution of the resulting equations, with careful attention to initial conditions, analytic expressions for the interior of the building shell, T_1, the temperatures of the room air, T_2, and the interior walls, T_3, are can be obtained, in response to a step heating input. The results of the analysis are applied to a specific building, and compared to limited data available from the literature.

THE THREE NODE MODEL

The three node capacitance-resistance network, shown in Figure 1, is used to study the dynamic response of a building to the input of heat. The thermal mass within the insulating envelope is divided into three portions: C_1, C_2, and C_3.

C_1 represents the thermal mass of the interior surface of the exterior wall at a temperature T_1. For a reasonably well insulated structure, most of the temperature drop to the outside is across the insulating layer. For short time periods the temperature dependence of the interior surface is dominated by the thermal conductance to the interior air, and not by conductance through the wall to the outside.

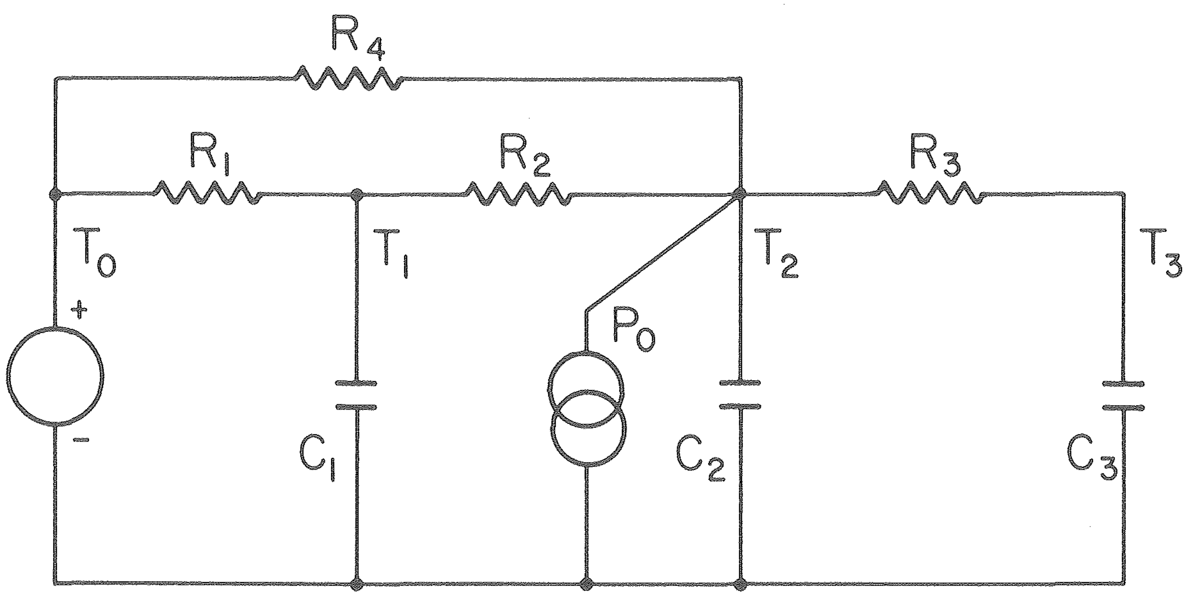
C_2 represents the thermal mass of the air enclosed within the volume of the residence at a temperature T_2. It is assumed in this analysis that air within the space is well mixed and is at a uniform temperature.

C_3 represents the thermal mass of the interior, walls at a temperature, T_3. For a passive solar structure this would also include the indirectly heated thermal mass.

There are four heat transfer paths represented as resistances: R_1 R_2, R_3, and R_4. Each thermal resistance is the reciprocal of the effective UA calculated for each path.

R_1 represents the resistance to heat loss from the interior surface of the external building shell at temperature T_1 to the outside temperature T_o.

R_2 represents the resistance to heat transfer from the air in the space at temperature T_2 to the interior surface at temperature T_1. This is approximately equivalent to the air film resistance of the inner surface



XBL 801-6603

Figure 1. The thermal network used to predict temperature changes in response to heat input.

of the building shell not counting windows.

Similarly R_3 represents the resistance to heat transfer from the air at temperature T_2 to the interior structure at temperature T_3.

Finally, R_4 represents the resistance to heat loss through the windows and by infiltration.

The forcing functions driving the system are the outdoor temperature, T_o and the heat input power $P(t) = P_o$, from the fan coil or furnace ductwork which transfers energy directly to the air within the space. Direct solar gain is not included in the analysis. Both the outdoor temperature T_o and the power delivered to the load are assumed to be constant over the period of the analysis corresponding to one thermostat on period or off period. By careful treatment of initial condition the analysis is reinitialized when the thermostat control states change.

THE EQUATIONS

The equations for the temperatures, T_1, T_2, and T_3, determined from the resistance capacitance network can be solved using the Kirchhoff analysis, where the currents represent heat flows and the voltages represent temperature. Laplace transform of these equations are then performed with careful consideration of the initial conditions at time t = 0; $T_1(0)$, $T_2(0)$, and $T_3(0)$. Assuming that the heat input, P_o, and outdoor temperature, T_o, are constant:

$$P(s) = \frac{P_o}{s} \quad ; \; T_o(s) = \frac{T_o}{s} \quad .$$

The Laplace transform equations are:

$$\frac{T_1(s)-T_o/s}{R_1} + \frac{T_1(s)-T_2(s)}{R_2} + C_1[sT_1(s)-T_1(0)] = 0$$

$$\frac{P_o}{s} = \frac{T_2(s)-T_1(s)}{R_2} + \frac{T_2(s)-T_o/s}{R_4} + \frac{T_2(s)-T_3(s)}{R_3} + C_2[s\,T_2(s)-T_2(0)]$$

$$\frac{T_3(s) - T_2(s)}{R_3} + C_3\,[s\,T_3(s) - T_3(0)] = 0$$

These equations can be put in matrix representation.

$$\begin{vmatrix} C_1(s + 1/t_1) & -1/R_2 & 0 \\ -1/R_2 & C_2(s + 1/t_2) & -1/R_3 \\ 0 & -1/R_3 & C_3(s + 1/t_3) \end{vmatrix} \begin{pmatrix} T_1(s) \\ T_2(s) \\ T_3(s) \end{pmatrix}$$

$$= \begin{pmatrix} C_1 \; T_1(0) + \dfrac{T_o}{s\;R_1} \\ C_2 \; T_2(0) + \dfrac{P_o}{s} + \dfrac{T_o}{s\;R_4} \\ C_3 \; T_3(0) \end{pmatrix}$$

where t_1, t_2, and t_3 are constants:

$$t_1 = \frac{C_1}{1/R_1 + 1/R_2}$$

$$t_2 = \frac{C_2}{1/R_2 + 1/R_3 + 1/R_4}$$

$$t_3 = R_3 C_3$$

ROOTS OF THE EQUATIONS

The characteristic time constants, τ_1, τ_2, and τ_3, that describe the dynamic response of the building temperatures are obtained from the roots of the determinant by solving the cubic equation:

$$\frac{t_2 \; (1 + st_1)}{R_3 \; C_2} + \frac{t_1 t_2 \; (1 + st_3)}{R_2 \; C_2 \; R_2 \; C_1} - (1+st_1)(1+st_2)(1+st_3) = 0$$

Building Parameters.

The characteristic time constants have been determined for two specific examples: a well insulated and an uninsulated residential structure. The building parameters for these two cases are shown in Table 1.

	BTU/hr-^oF				
	$1/R_1$	$1/R_2$	$1/R_3$	$1/R_4$	$1/R_{eff}$
well insulated	243	3111	653	279	504
uninsulated	1317	3111	653	629	1554

	BTU/^oF		
	C_1	C_2	C_3
well insulated	2470	245	1036
uninsulated	2470	245	1036

Table 1. Building Parameters

The building modeled is a 1700 ft^2 (158 m^2) residence treated as a single zone. The well insulated structure has R-values (in ft^2-hr-^oF/BTU) of 17 (floor), 32 (ceiling), and 21 (walls) with 200 ft^2 (12 % of floor area) (19 m^2) double glazed windows and an infiltration rate of 2/3 air change per hour. The uninsulated structure has R-values of 5.4(floor), 5.2 (ceiling), and 4.4 (wall) typical of frame construction with 340 ft^2 windows (20 % of floor area) and an infiltration rate of one air change per hour. From these values the thermal resistance terms are calculated.

The effective steady state heat loss rate from the air in the space to the outside is given by the series resistances R_1 and R_2 in parallel with R_4.

$$1/R_{eff} = 1/R_4 + 1/(R_1 + R_2)$$

The thermal masses in the space are estimated:

1) C_1, the thermal equivalent of 1/2 inch of sheet rock on the inner surface of the exterior walls, ceiling, and floor; 2) C_2, the thermal mass of the building air volume; and 3) C_3, the estimated thermal mass of 960 ft^2 of interior partition walls.

Time Constants.

The exact time constants predicted by the model for the building require the solution for the roots of the cubic equation and are shown in Table 2.

Typical	τ_1	τ_2	τ_3
well insulated	0.057	1.29	7.72
uninsulated	0.053	0.97	2.73
Experimental*			
minimum	0.098	0.50	10.3
maximum	0.066	0.43	6.6

Table 2. Time constants in hours. *Observed data is from Socolow and Sonderegger[1] for a Twin Rivers townhouse.

The longest time constant, τ_3, which describes the relaxation of the total building thermal mass at the effective heat loss rate, $1/R_{eff}$, is approximately given by

$$\tau_3 \cong [\, C_1 + C_2 + C_3]\, R_{eff}$$

The approximate value of τ_3 is 7.44 hrs (exact root 7.72 hrs) for the insulated case and 2.41 hrs (exact root 2.73 hrs) for the uninsulated case. The longest time constant depends both on the thermal mass in the space and the effective heat loss rate.

The shortest time constant τ_1 is related to the relaxation of the space air temperature by heat loss to the walls and to the outside through the windows and by infiltration. τ_1 is given approximately by;

$$\tau_1 \cong C_2 \, [1/R_2 + 1/R_3 + 1/R_4 \,]^{-1}$$

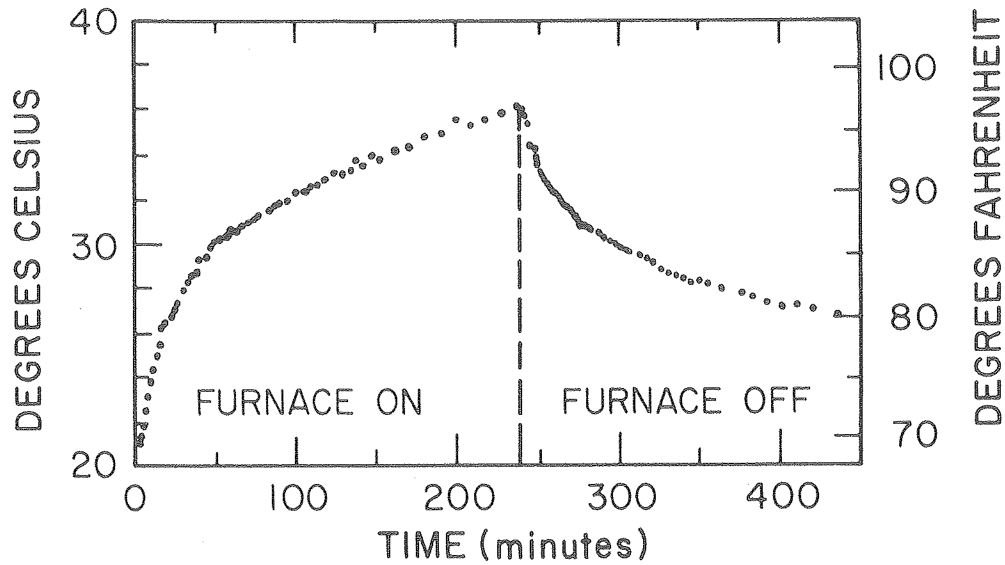
The approximate value for t_1 is 0.061 hrs (exact root 0.057) for the insulated case and 0.056 hrs (exact root 0.053) for the uninsulated case. The shortest time constant is sensitive to the thermal capacitance of the air and the coupling to the thermal mass in the space and does not change significantly with degree of insulation of the space.

The intermediate time constant τ_2 is related to the redistribution of energy within the space and is not readily identified.

Experimental Observation.

Socolow and Sonderegger[1] have analysed a single experiment in which a building, different from the one modeled, was "toasted" with the furnace on for 4 hours and then allowed to cool for 3.5 hours while the temperature in a hallway is observed. The hallway temperature rose from from 68 ^{o}F (20 ^{o}C) to 97 ^{o}F (36 ^{o}C) and then cooled. Their observations of the hall temperature are shown in Figure 2.

Subsequent analysis of this experiment revealed three distinct time constants for the heating and cooling process. Because of limited data, their time constants depend somewhat on assumptions made as to the final temperature reached, so that they were able to give only the range of values that are shown in Table 2. As we have little information about the conditions of the experiment not too much can be made from their results. However, their measured short and long time constants are certainly close to the values obtained for the well insulated structure,



(a) HALL TEMPERATURE VERSUS TIME

XBL 801 · 6604

Figure 2. Hallway temperature as a function of time. Adapted from Socolow and Sonderegger[1].

and the intermediate time constant is the correct order of magnitude.

TIME RESPONSE

The matrix equations are solved using Cramer's rule to determine the time response of the room air, $T_2(s)$, and wall temperatures, $T_1(s)$ and $T_3(s)$. Here only the solution for the room air temperature, T_2, will be presented, but the others are similar. The Laplace transform for the room air temperature is given by

$$T_2(s) = \frac{1}{|\det|} \begin{vmatrix} (1/t_1 + s)C_1 & C_1\ T_1(0) + T_o/sR_1 & 0 \\ -1/R_2 & C_2\ T_2(0) + \dfrac{P_o}{s} + \dfrac{T_o}{s\ R_4} & \dfrac{-1}{R_3} \\ 0 & C_3\ T_3(0) & (1/t_3 + s)C_3 \end{vmatrix}$$

where $T_1(0)$, $T_2(0)$, and $T_3(0)$ represent the initial temperatures at time $t=0$. The value of the determinant is given by

$$|\det| = \frac{C_1\ C_2\ C_3}{t_1\ t_2\ t_3}\ F'(1 + s\tau_1)(1 + s\tau_2)(1 + s\tau_3)$$

where the constant F' is given by

$$F' = 1 - \frac{t_1\ t_2}{R_2{}^2 C_1 C_2} - \frac{t_2}{R_3 C_2} = \frac{t_2(R_1 + R_2 + R_4)}{C_2 R_4 (R_1 + R_2)}$$

The solution for the Laplace transform of the room air temperature is given by:

$$\begin{aligned} T_2(s) = \Bigg\{ & \frac{t_2(1 + st_1)(1 + st_3)}{F'C_2}\left(\frac{P_o}{s} + \frac{T_o}{s\ R_4}\right) \\ & + \frac{t_1 t_2(1 + st_3)}{F'C_1 C_2 R_1 R_2}\ \frac{T_o}{s} + \frac{t_1 t_2(1 + st_3)}{F'C_2 R_2}\ T_1(0) \\ & + \frac{t_2(1 + st_1)(1 + st_3)}{F'}\ T_2(0) + \frac{t_2 t_3(1 + st_1)}{F'C_2 R_3}\ T_3(0) \Bigg\} \\ & \times \frac{1}{(1 + s\tau_1)(1 + s\tau_2)(1 + s\tau_3)} \end{aligned}$$

Transforming into the time domain under the assumption that the initial temperatures are equal, $T_1(0) = T_2(0) = T_3(0) = T_{in}(0)$, the solution takes on a (relatively) simple form. For the well insulated case;

$$\begin{aligned} T_2(t) = {} & P_o R_{eff} + T_o \\ & + \{ -0.11\ P_o R_{eff} - 0.06\ [T_o - T_{in}(0)] \}\ e^{-t/\tau_1} \\ & + \{ -0.02\ P_o R_{eff} - 0.03\ [T_o - T_{in}(0)] \}\ e^{-t/\tau_2} \\ & + \{ -0.86\ P_o R_{eff} - 0.91\ [T_o - T_{in}(0)] \}\ e^{-t/\tau_3} \end{aligned}$$

After a long time, $t \gg \tau_3$, the solution reduces to the steady state temperature.

The time dependent temperature equation can be put in a general form:

$$T_2(t) = P_o R_{eff} + T_o$$
$$- \sum_j \{ w_{hj} P_o R_{eff} + w_{cj} [T_o - T_{in}(0)] \} e^{-t/\tau_j}$$

where w_{hj} and w_{cj} are the heating and cooling weighting factors for each time constant τ_j.

The calculated weighting factors for the well insulated and uninsulated cases, as well as for the observed values by Socolow and Sonderegger[1] are shown in Table 3. Without specific knowledge of the experimental measurements, comparison is difficult. However, the weighting factors calculated by our simple model for both the heating and cooling cases, bracket the values given by the experiment.

	Heating			Cooling		
	w_{h1}	w_{h2}	w_{h3}	w_{c1}	w_{c2}	w_{c3}
well insulated	0.10	0.02	0.86	0.06	0.03	0.91
uninsulated	0.32	0.16	0.53	0.11	0.28	0.61
observed	0.09	0.14	0.76	0.10	0.20	0.70

Table 3. Heating and cooling weighting factors.

SENSITIVITY ANALYSIS

Sensitivity studies were run to observe the dependence of the time constants and the weighting factors on different parameters. Comparison of different cases where the infiltration and glazing loss rates, degree of insulation, and distribution of thermal mass are varied are shown in

Table 4.

	BTU/hr-^oF			hours		
	$1/R_1$	$1/R_4$	$1/R_{eff}$	τ_1	τ_2	τ_3
Infiltration	243	279	504	0.057	1.29	7.72
	243	450	675	0.055	1.28	6.03
	243	629	860	0.053	1.27	4.94
Insulation	243	376	601	0.056	1.29	7.41
	750	376	980	0.056	1.15	3.96
	1317	376	1301	0.056	0.99	3.02

	BTU/^oF		hours		
	C_1	C_3	τ_1	τ_2	τ_3
Capacitance	3000	500	0.057	0.79	7.41
	2000	1500	0.056	1.43	8.28
	1000	2500	0.052	1.04	10.2

Table 4. Sensitivity studies. Results for several different cases. In all studies the thermal mass of the air, C_2, and the transfer from air to wall, $1/R_2$ and $1/R_3$, were held constant.

Infiltration.

If the value of the infiltration and glazing loss parameter, $1/R_4$, is increased from 279 BTU/hr-^oF for 200 ft^2 of double glazing with 2/3 air change per hour, to 629 for 340 ft^2 of single glazing with one air change per hour, the short time constants, τ_1 and τ_2, change only by a few percent indicating that the internal rearrangement of heat is unaffected. However, the long time constant, τ_3, decreases from 7.72 hrs to 4.9 hrs as expected because of the increased loss from the thermal mass.

Insulation.

As the insulation value of the wall is decreased from heavy to minimal insulation, the short time constant is unchanged. The intermediate time constant decreases by 30 % and the long time constant falls by a factor of two. The weighting factors, shown in Table 3, also indicate that the importance of the intermediate time constant increases with decreasing insulation.

Capacitance.

If the thermal mass within the structure, $C_1 + C_2 + C_3$, is kept constant, but is shifted from the exterior wall to the interior of the space, the short time constant is almost unchanged, the intermediate time constant becomes somewhat larger, and the long time constant goes from 7.4 hrs to 10.18 hours as the thermal mass is moved further from the exterior walls.

CONCLUSIONS

This simplified building load model predicts thermal behavior of typical buildings based on physically definable building parameters. The temperature response of a building to a change in heat input, can be characterized in terms of an effective energy loss term, R_{eff}, three heating weighting factors w_{hj}, and three cooling weighting factors, w_{cj}, and three time constants, τ_j, which are straightforward to evaluate from building parameters and which are experimentally observable. The model gives predictions which agree in general with those observed in an experiment with a real building.

It is hoped that this paper will prompt interest in careful observation and evaluation of short time constants of buildings. These short time dynamics are important in implementing building temperature control strategies.

The wall and air temperatures predicted by this model can serve as forcing functions for a thermostat response model to determine when heating is required. In our analysis we have not included transit delays and heating delivery time constants, although extension of the analysis to include them is straightforward. With a active solar heating system, the rate of heat delivery depends on the storage tank temperature, so that the thermostat cycling time is not a simple function of the indoor-outdoor temperature difference. This three time constant model may also serve to model the control response of passive structures to back-up heating systems.

REFERENCES AND FOOTNOTES

[1] Robert H. Socolow and Robert C. Sonderegger, "The Twin Rivers Program on Energy Conservation in Housing: Four Year Summary Report," Center for Environmental Studies, Report No.32, August 1976, Princeton University. Excerpts in ASHRAE Trans., 83(1), (1977).

[2] Mashuri L. Warren, Steven R. Schiller, and Michael Wahlig, "Experimental test facility for evaluation of controls and control strategies," Second Annual Systems Simulation and Economic Analysis Conference, 23-25 January, 1980, San Diego, CA

[3] M. Majteles, H. Lee, M. Wahlig and M. Warren, "Experimental Test Facility for Evaluation of Solar Control Strategies," Proc. Workshop on Control of Solar Energy Systems for Heating and Cooling. Hyannis, Mass. 23-25 May, 1978. (LBL-8308) Aug. 78.

[4] W. K. Roots, M. Shridhar, R. H. Tull, and J. R. Pfafflin, "Mode-

Dependent Time Constants in Three Forms of Space Heating," IEEE Transactions on Industry Applications, Vol. 1A-10, No. 6 (Nov/Dec 1974), pp. 699-702

[5] R. C. Sonderegger, "Dynamic Models of House Heating Based on Equivalent Thermal Parameters," Princeton University, Center for Environmental Studies, Report PU/CES 57 (September 1977).

[6] R. O. Zermuehln, H. L. Harrison, "Room Temperature Response to a Sudden Heat Disturbance Input," ASHRAE Transactions, 71, (1965), pp. 206-210.

[7] H. L. Harrison, W. S. Hansen, and R. E. Zelenski, "Development of a Room Transfer Function Model for Use in the Study of Short-term Transient Response," ASHRAE Transactions, 74, (1968), pp.198-206.

[8] K. M. Letherman, "Functional Modeling of Room Temperature Response and Effects of Thermostat Characteristics on Room Temperature Control," in C. J. Hoogendoorn and N.H. Afgan, Energy Conservation in heating, cooling, and ventilating buildings (Washington:Hemisphere Publishing Corp., 1978), pp.393-404

[9] A. V. Sebald, F. Langenbacher, and J. R. Clinton, "On Proper Control of Direct Gain Solar Houses," Proc. 4th National Passive Conference, AS/ISES, 3-5 October 1979,

[10] A. Kaya, "Analytic Techniques for Controller Design," ASHRAE Journal, April 1976, pp. 35-39.